The statement that 'the language of science is mathematics' is attributed to Galileo. Was he right?

Curiosity has consistently driven humanity to search for answers on the workings of the world around us and the reasons why things behave how they do. In the immense complexity facing us, we began to uncover a simple underlying logic hidden throughout nature that helps us to make sense of the universe – mathematics.

Science is fundamentally the study of patterns and relationships in the natural world through observations and experiments, and mathematics provides an effective and concise means of expressing patterns and relationships – so it is clear to see why mathematics would be a useful tool for describing scientific ideas. But before concluding whether it can be given the title of 'the language of science', we must first define the meaning of a language. According to the Oxford dictionary, a language is a "system of communication used by a particular country or community" [1] – this essay will therefore explore the various ways in which mathematics is used to explain and communicate ideas within the scientific community.

Physical Sciences

Two areas of science which rely heavily on mathematics to communicate their ideas and theories are astrophysics and quantum mechanics, as experiments to demonstrate these ideas are often impractical or impossible to execute due to the macroscopic and microscopic scales.

For example, relativistic effects such as time dilation or length contraction have been observed through science and quantified using mathematics. Evidence for particles 'living longer' when travelling at speeds close to the speed of light has been demonstrated in several experiments in which particles are accelerated – when the very short-lived muon particle travels at 99.92% of the speed of light, it lives roughly 25 times longer and travels 25 times further than it theoretically should. Similar experiments exploring this relativity-induced time travel at CERN have allowed the even shorter-lived phi meson to experience a γ factor (see below) of around 5,000 whilst traveling at 99.99% the speed of light.

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

The Lorentz factor

The extent of this length contraction and time dilation is given by the Lorentz factor, named after the physicist Hendrik Lorentz who defined the mathematical equations clarifying the connection between time and distance, when one frame of reference moves at a constant speed relative to another.

This mathematical equation can then be used to demonstrate how the magnitude of this effect increases exponentially as the speed of the particle approaches the speed of light.

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{1}{4}\right)^2}} = 1.03 \text{ (3sf)}$$

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{1}{2}\right)^2}} = 1.15 \text{ (3sf)}$$

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{99}{100}\right)^2}} = 7.09 \text{ (3sf)}$$

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{99.999}{100}\right)^2}} = 224 \text{ (3sf)}$$

Expressing the scientific observations made in particle accelerators as a mathematical equation is a clear and effective way of explaining the relationship between the particle's speed and the extent of time dilation/length contraction that it experiences, and makes it easy to communicate to others the reason why we do not experience these counterintuitive relativistic effects in our everyday lives.

Chemical Sciences

Mathematics also plays a preponderant role in the chemical sciences; we use logarithms to find the concentration of hydrogen ions in solution and carry out calculations to find the purity of samples or the percentage yield, which is highly useful in chemical industries such as pharmaceutical manufacturing.

Chemists also utilise mathematical formulae to determine the rate of chemical reactions. A chemical reaction is defined as the change in concentration of a reactant or product over the change in time. [6] Chemists can easily observe how, as the concentration of a reactant is increased, the initial rate of the reaction increases. However, mathematics allows us to precisely define this relationship. Using the decomposition of dinitrogen pentoxide as an example, the ratio for the rate at which each reagent and product reacts is given by the following formula:

$$2N_2O_{5\,(g)} \longrightarrow 4NO_{2\,(g)} + 1O_{2\,(g)}$$

$$\text{Rate}: \frac{1}{1}\frac{\Delta[O_2]}{\Delta t} = \frac{1}{4}\frac{\Delta[NO_2]}{\Delta t} = -\frac{1}{2}\frac{\Delta[N_2O_5]}{\Delta t}$$

The numbers in red highlight how the way in which the average reaction rate is expressed is linked to the amount, in moles, of each reactant and product.

Consider the general reaction $aA + bB \longrightarrow \text{products}$ where 'a' and 'b' represent the number of moles of reagents 'A' and 'B' respectively. The rate of this reaction is given by the general rate law:

$$R = k[A]^x[B]^y$$

where 'R' is the rate of the reaction, 'k' is the rate constant (unaffected by the concentrations of A and B, though it does depend on the temperature) and '[A]' and '[B]' are the rates of reaction of the reagents A and B. The orders of the rate of reaction for reactants A and B cannot be calculated using the equation for the reaction, so 'x' and 'y' must be determined experimentally. This can be done by keeping one of the reactants at a constant concentration whilst varying the other and recording the initial reaction rate.

[A]	Initial Rate	
1.00 M	0.01 M/s	$R \propto [A]^1$
2.00 M	0.02 M/s	
3.00 M	0.03 M/s	

[B]	Initial Rate	
1.00 M	0.01 M/s	$R \propto [B]^2$
2.00 M	0.04 M/s	
3.00 M	0.09 M/s	

For example, if this were the data obtained for the experiment, the rate of reaction could then be neatly expressed using the mathematical formula:

$$R = k[A]^1[B]^2$$

We can then substitute a set of values into the rate formula to determine the rate constant k. Once the rate constant is known, the mathematical formula can be reused to accurately predict the rate of this reaction if the concentrations of the reactants were changed.

Calculus can then be used to determine the integrated rate law for reactions of a particular order, to express the concentrations of the products or reactants as a function of time. [7]

This is just one example of how mathematics is used in chemistry. It is particularly interesting to note that experimental data was required in order to originally determine the mathematical relationship between the concentrations and reaction rate – the science is needed to observe and record the trend in the data, and the mathematics can then communicate these patterns and relationships in a concise and accurate manner.

Medical sciences

Of all the major scientific fields that use mathematics, medicine is one of the least intuitive; yet this sector makes use of a surprisingly high content of mathematics to communicate a large range of concepts, from using radon transforms to reconstruct images from MRI scans, to the intricacies of designing a heart bypass using the laws of fluid dynamics.

A current area of research is proton therapy, an advanced form of radiation therapy that fires a beam of protons at the tumour, rather than high-energy photons (x-rays or gamma rays). Both methods cause damage to DNA by ionising atoms and molecules within the cells, but neither are able to distinguish healthy cells from cancerous ones, which may result in further complications. This is where proton therapy has a significant advantage over conventional radiotherapy; both types of radiation must pass through healthy tissue to reach the tumour, causing damage to surrounding cells. However, whilst photons deposit a gradually decreasing amount of energy as they travel, protons release a relatively low amount of energy on their journey to the tumour and then deposit a significant proportion of their energy upon reaching it, causing a relatively large amount of damage to the cancerous cells and minimal damage to the healthy cells.

This behaviour of protons can be explained using mathematics. In 1903, the scientist William Bragg investigated the rate at which charged particles lose energy whilst travelling through various materials. It was shown that the rate of energy loss over a distance is inversely proportional to the proton's velocity squared.

$$\text{Rate of energy loss with distance} \propto \frac{1}{\text{speed}^2}$$

$$\frac{dE}{dx} \propto \frac{1}{V^2}$$

When the particle's velocity is low, the fraction $1/v^2$ becomes larger, so the rate of energy loss increases exponentially as the particle is slowed down.

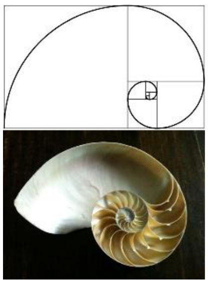
The mathematics therefore allows us to accurately calculate the optimal kinetic energy at which to release the protons so that there are a series of Bragg peaks at the tumour's exact location.

Whereas the use of new technologies such as these are sometimes debated over, mathematical equations provide scientists with a way to demonstrate and explain the effects and advantages in an indisputable way. [9]

Biological Sciences

Another area of science that one wouldn't initially expect to use a great deal of mathematics to communicate its ideas is biology. However, mathematics has proven itself to be an equally indispensable tool in this field; whether using exponential functions to model the population dynamics of a species, calculating the frequencies of gene expression, or using vector calculations to predict the future migration paths of a species, mathematics plays a fundamental role. [10]

Mathematical patterns are often found in the natural world, from the symmetry of snowflakes to the fractals found in trees. A well-known example is the golden ratio and Fibonacci sequence, found throughout nature, from DNA at the microscopic scale to spiral galaxies at the other extreme, as well as in our everyday lives in pinecones, flowers and shells.



The Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55...

$$\varphi = 1.618\ldots \qquad \frac{34}{21} = 1.619\ldots \qquad \frac{55}{34} = 1.618\ldots$$

The golden ratio, represented by the Greek letter phi, is an irrational number obtained by dividing consecutive pairs of Fibonacci numbers, with approximations becoming increasingly accurate along the sequence.

This ratio provides an evolutionary advantage, as having an irrational value ensures that leaves grow at non-repeating angles, therefore maximising their exposure to the light. [11]

Another mathematical pattern in nature was uncovered when biologists studied the migratory patterns of the cicadas – a type of insect found across North America – and found that they emerge periodically; usually every 13 or 17 years, but always a prime number of years. Though this pattern could be dismissed as a coincidence, the fact that prime numbers are indivisible by factors other than 1 and itself actually gives these insects a greater chance of survival, as their migration would coincide much less frequently with predators who also show periodic behaviour. Over time, they have therefore obtained this survival advantage though evolution. [12]

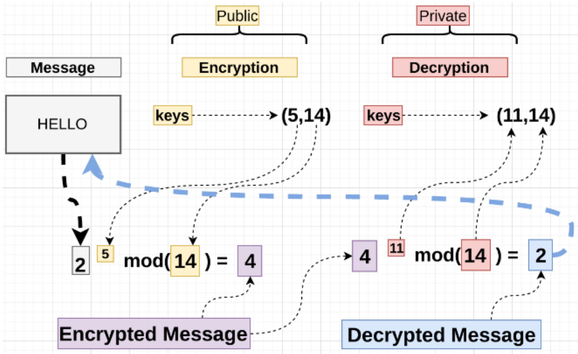
In each case, it is the scientists who observe and record these patterns in the natural world, but it is the underlying mathematics which has the ability to explain and communicate why these patterns exist.

Computer and Information Sciences

Mathematics is at the heart of almost every computer science discipline, playing a critical part in everything from augmented reality and online simulations to algorithms and coding.

A highly important area of computer science is data encryption, to ensure the security of our communication systems. A cryptography method that makes use of mathematics is public key encryption, which uses RSA algorithms. These are based on the idea that multiplying two prime numbers together is relatively easy, but factorising this semiprime to return to the original two prime factors is significantly more challenging; there is no known method except systematically testing numbers, and this can often require greater computational power than is currently available if the semiprime is large enough.

Here is a simplified version of how it works: two prime numbers, p and q, are multiplied together – this becomes the modulus.



Then a list is made from 1 to the modulus, and all common factors are removed – a quick way to do this is (q - 1)(p - 1). We are left with a list of length L. Then the encryption key is picked; it must be between 1 and L, as well as coprime with L and the modulus. To find the key for decryption, one must follow the rule that the decryption key multiplied by the Encryption key modulus the length of the non-common factor with the modulus must equal 1. [13]

So whereas mathematics can be used as a language to simplify and explain scientific concepts, it can conversely be used in computer science as an undecipherable language, helping to ensure our communication is encrypted and secure.

The language of the universe

It is evident that we make use of mathematics as a language of science in practically every field - and not just for the purpose of communicating ideas to the public or other members of the scientific community. Unlike any other language, mathematics is universal – whereas the science can be studied and expressed in various languages, the mathematics is constant no matter the country, continent or even galaxy – so much so that it is the only possible method of communication with extra-terrestrial intelligence, as far as we know. The most famous example of this is the Arecibo Message, an interstellar signal sent from earth to the star cluster M13 in 1974 with the aim of contacting ‘aliens’. The message, depicting basic information about humanity and life on Earth, such as the structure of DNA and the position of our planet in the solar system, used mathematics to broadcast these ideas in the simplest and most elegant way. It consisted of 1679 binary digits - this number was chosen as it is a semiprime (the product of two primes). Despite their fundamental nature, prime numbers present one of the greatest enigmas in mathematics due to their unpredictability – it has so far been impossible to spot a pattern within the order they come in. It is hoped that this number of digits would therefore prompt intelligent life to recognise the non-natural origin of the message.

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H C N O P
1 6 7 8 15
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0 0 0 1 1
0 1 1 0 1
0 1 1 0 1
1 0 1 0 1
X X X X X
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The binary numbers to represent the atomic numbers of hydrogen, carbon, nitrogen, oxygen, and phosphorus; the components of DNA.

By sending a message - in binary, with a semiprime number of digits – that depicts data such as our chemical make-up, mathematics is being exploited as a way of communicating life on earth, making it the language of science in its most literal sense.

Conclusion

Having seen just a few examples of how every scientific field uses mathematics to describe and communicate their findings, from Einstein's equations describing special relativity to the statistical models predicting biological events, it is not surprising that Galileo came to the conclusion that the language of science is mathematics – it could understandably be considered the purest and most efficient method of communication, devoid of ambiguity.

Mathematics is a precise and accurate way of expressing complex ideas, and it possesses an important quality that other methods of communications lack – it is indisputable. As well as Galileo, this was recognised by the renowned biological scientist J. B. S. Haldane, who once said that "(…) if you are faced by a difficulty or a controversy in science, an ounce of algebra is worth a ton of verbal argument".

Mathematics could be viewed as the framework of the language of science – not dissimilar to the structure that grammar provides to conventional languages. The mathematical equations and graphs come together with diagrams and explanations, to concisely and efficiently communicate to others the incredible breakthroughs and concepts developed within the scientific community.

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